BUG REPORT Talat Odman Georgia Tech June 1, 2009

Synopsis: In CMAQ, the Smagorinsky horizontal diffusion coefficient assumes Cartesian coordinates. This may introduce a directional bias to horizontal diffusion in map projection coordinates, especially when the map scales display a wide range over the modeling domain such as in hemispherical applications with polar stereographic coordinates.

Description: The original horizontal diffusion coefficient of Smagorinsky,

$$K_{h} \propto \sqrt{\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right)^{2} + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^{2}}$$
(1)

is valid in Cartesian coordinates. Using this parameterization in other coordinate systems should have interesting effects.

Remedy: Becker and Burkhardt $(2007)^1$ revisited Smogarinsky's mixing-length based parameterization of horizontal diffusion and modified it for spherical and terrainfollowing vertical coordinates used in general circulation models. We are in the process of doing the same for the coordinates used in regional-scale atmospheric modeling. Until we come up with a more rigorous derivation, I am proposing the following form that I obtained by intuition, using the similarity of the right-hand-side in Equation (1) to divergence.

$$K_{h} \propto \frac{1}{\sqrt{\gamma}} \sqrt{\left(\frac{\partial}{\partial x^{1}} \left(\sqrt{\gamma} u^{1}\right) - \frac{\partial}{\partial x^{2}} \left(\sqrt{\gamma} u^{2}\right)\right)^{2} + \left(\frac{\partial}{\partial x^{1}} \left(\sqrt{\gamma} u^{2}\right) + \frac{\partial}{\partial x^{2}} \left(\sqrt{\gamma} u^{1}\right)\right)^{2}}$$
(2)

In map projection coordinates used in CMAQ, where *m* is the map scale factor,

 $\sqrt{\gamma} = \frac{1}{m^2}$; $x^1 = x_m$; $x^2 = y_m$; $u^1 = mU$; and $u^2 = mV$. Therefore,

$$K_{h} \propto m^{2} \sqrt{\left(\frac{\partial}{\partial x_{m}}\left(\frac{U}{m}\right) - \frac{\partial}{\partial y_{m}}\left(\frac{V}{m}\right)\right)^{2} + \left(\frac{\partial}{\partial x_{m}}\left(\frac{V}{m}\right) + \frac{\partial}{\partial y_{m}}\left(\frac{U}{m}\right)\right)^{2}}$$
(3)

I am currently analyzing the difference of Equations 1 and 3 on polar stereographic coordinates.

¹ Becker, E. and U. Burkhardt (2007): Nonlinear Horizontal Diffusion for GCMs, *Monthly Weather Review*, **135**, 1439-1454.